Exercise 2.4.3

Use linear stability analysis to classify the fixed points of the following systems. If linear stability analysis fails because $f'(x^*) = 0$, use a graphical argument to decide the stability.

$$\dot{x} = \tan x$$

Solution

The fixed points occur where $\dot{x} = 0$.

$$\tan x^* = 0$$

$$x^* = n\pi, \quad n = 0, \pm 1, \pm 2, \ldots$$

Use linear stability analysis to classify these points.

$$f(x) = \tan x$$

Differentiate $f(x)$.

$$f'(x) = \sec^2 x$$

$$= \frac{1}{\cos^2 x}$$

As a result,

$$f'(n\pi) = \frac{1}{(\cos n\pi)^2} = \frac{1}{(\pm 1)^2} = 1 > 0 \quad \Rightarrow \quad x^* = n\pi \text{ are unstable fixed points.}$$

The graph of $\dot{x}$ versus x confirms this.